Geometric note layouts for steelpans and other polyhedral idiophones

The steelpan of Trinidad and Tobago is the only acoustic instrument invented in the twentieth century, tracing its origins to the surplus of oil drums left in the wake of WWII. Recent years have seen the invention of other instruments, such as the Swiss-made Hang, that also feature a curved, metal surface fashioned to sound multiple pitches; together with the steelpan, these may best be described as polyhedral idiophones. As experiments to their structural designs continue, this paper offers a method of deriving geometric note layouts for such instruments.

I begin by representing the note layout as a layer of circles densely packed with triangular interstices, with the area of each circle directly proportional to its pitch frequency. Ease of performance calls for notes related by octave to be aligned, while acoustics favors placing harmonically distant notes further apart. The circle of fifths, upon which the layout of the tenor steelpan is based, proves ideal in this sense. However, I also consider the possibility of a semitonal arrangement; this can be accommodated by the recently designed bore pan, which keeps notes acoustically isolated (Figure 1).

Each note layout is generated by a "flower," a single circle encompassed by neighboring circles, called "petals." Three pairs of opposite petals are equidistant to the center circle, each by a single interval approached from opposite directions: an octave, a semitone *or* a fifth, and some combination of these two intervals (Figure 2). Extending the flower creates a Euclidean surface that wraps back onto itself and spirals towards an apex like the lateral surface of a cone (Figure 3). I use the law of cosines to confirm two observations: first, that the angle sum of a flower is always 360 degrees (Figure 4); and second, that the rotation angle owes to the difference in size between the flower's two non-octave intervals, with smaller differences leading to more evenly sized petals and narrower cones (Figure 5).

Finally, I consider the possibility of ovoid notes (Figure 6); this would result in irregular overtones such as those that contribute to the steelpan's distinctive timbre. Gradually lengthening the antipodal distance of each ovoid as the pitch increases would allow the surface of the instrument to be spheroid rather than conical.

As a side note, the geometry used in music theory tends to be primarily concerned with discrete functions; this challenge of designing acoustic instruments presents a unique opportunity for music theorists to work with continuous relations. I will end the paper by addressing feedback from various steelpan crafters and performers, which might be of interest to ethnomusicologists.

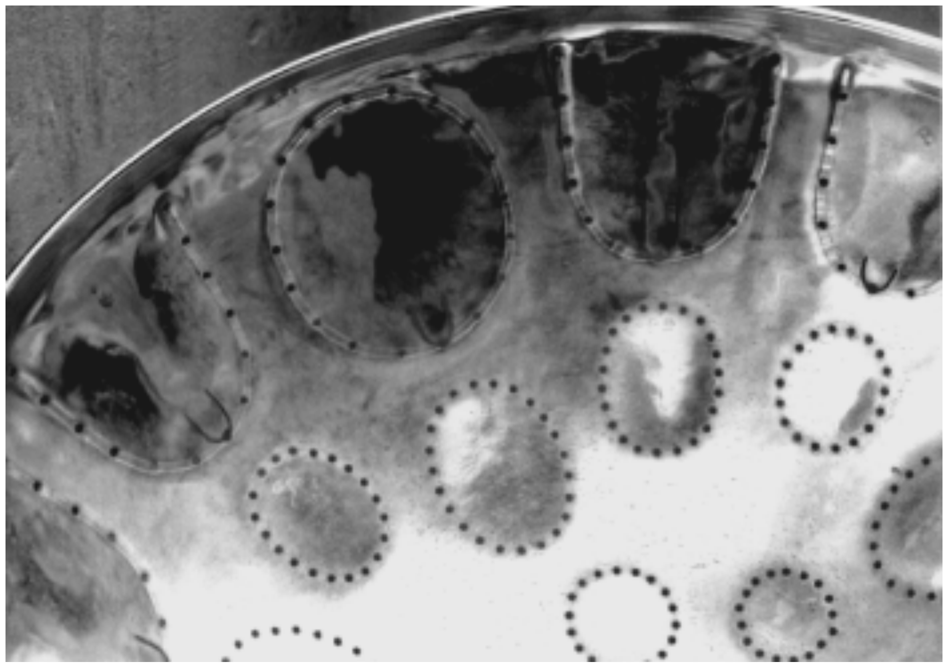


Figure 1. Denzil Fernandez's bore pan design

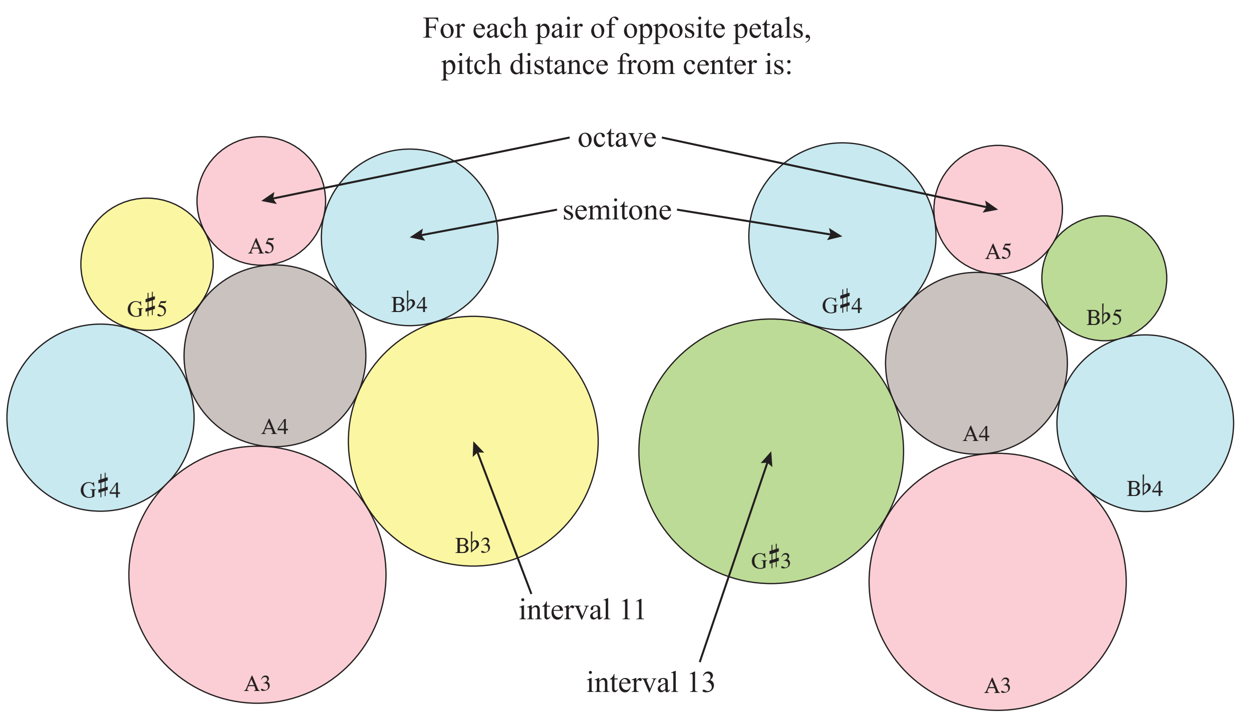
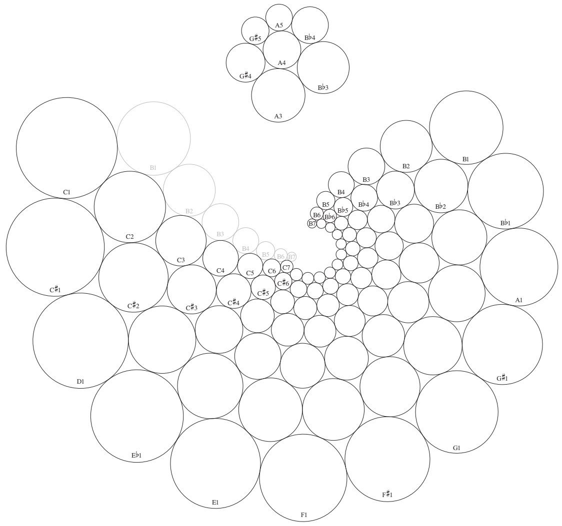
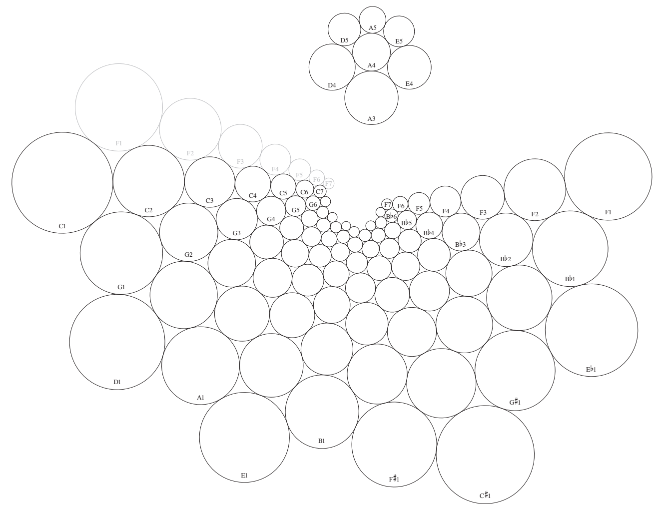


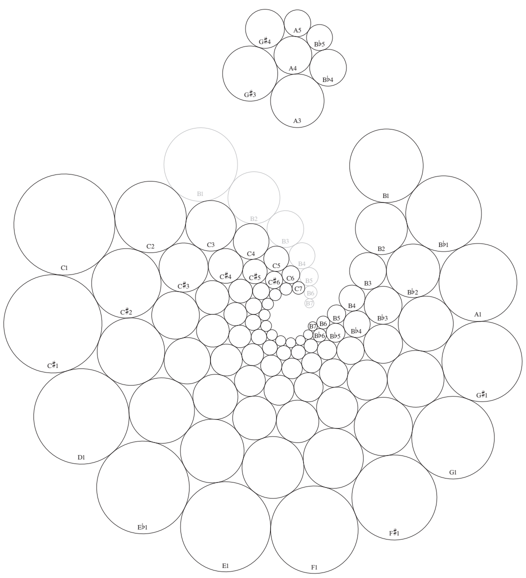
Figure 2. Intervals represented by pairs of opposite petals



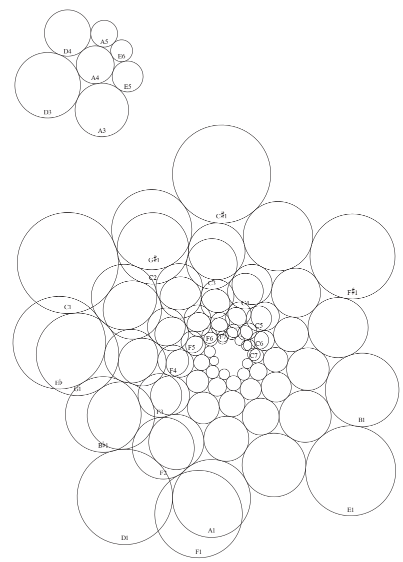
semitone and interval 11



fifth and fourth



semitone and interval 13



fifth and interval 17

Figure 3. Some note layouts generated by flowers, each labeled by its two non-octave intervals

Given area of a circle $A = \pi r^2$, each petal radius is the square root of that petal's frequency relative to the pitch of the center circle, whose radius here is 1.

Then, given sides of red triangle with lengths $a = r_0 + r_1$, $b = r_0 + r_2$, and $c = r_1 + r_2$, its angles may be determined using the law of cosines $c^2 = a^2 + b^2 - 2ab\cos\gamma$.

The flower angle sum will always be 360 degrees, confirming its Euclidean surface.

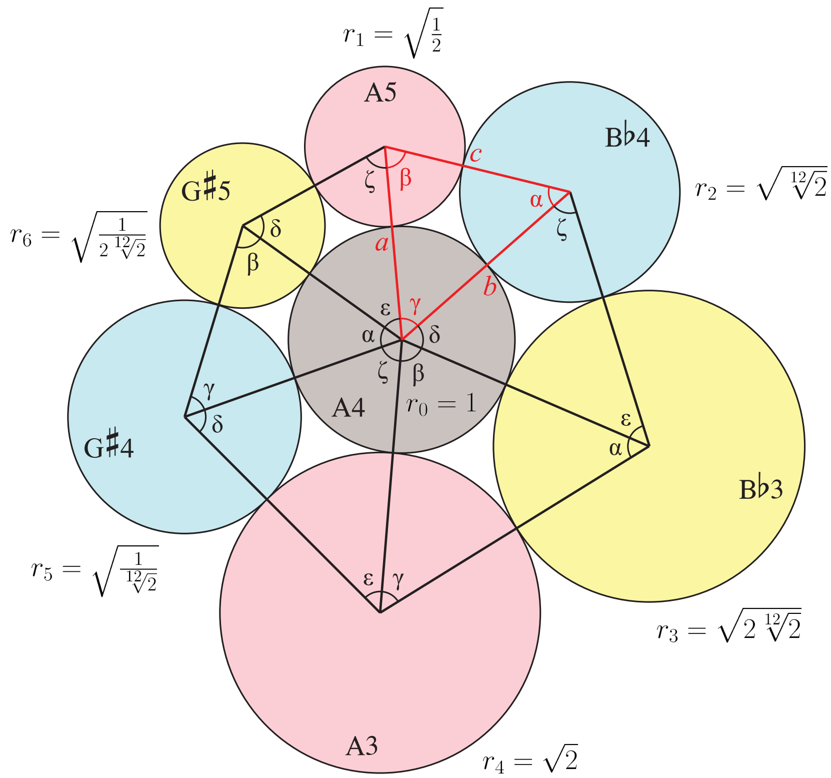


Figure 4. The flower's Euclidean surface

Segments and angles in black have known values.

Segments and angles in red may be solved for by using law of cosines.

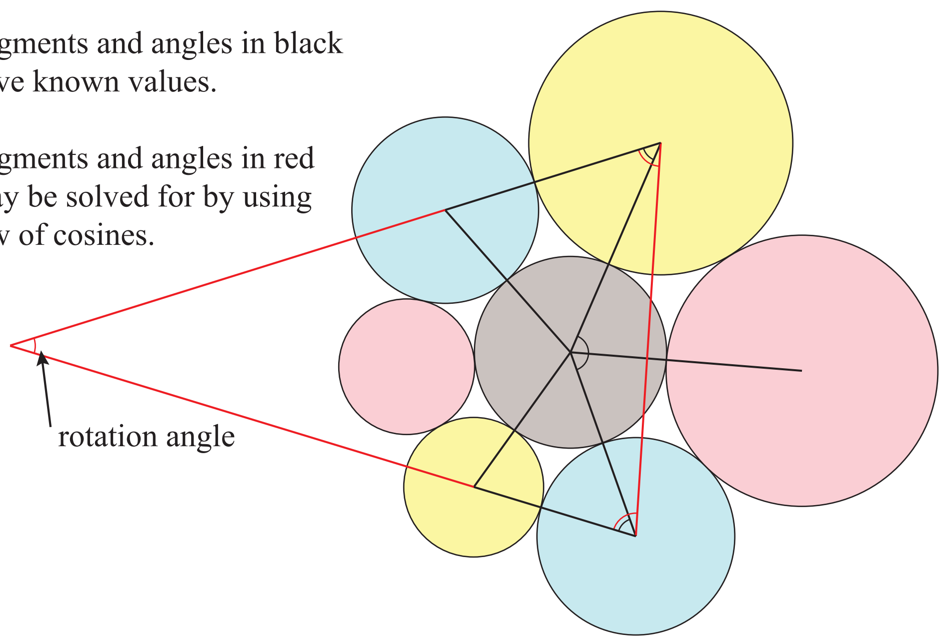


Figure 5. Measuring the flower's rotation angle

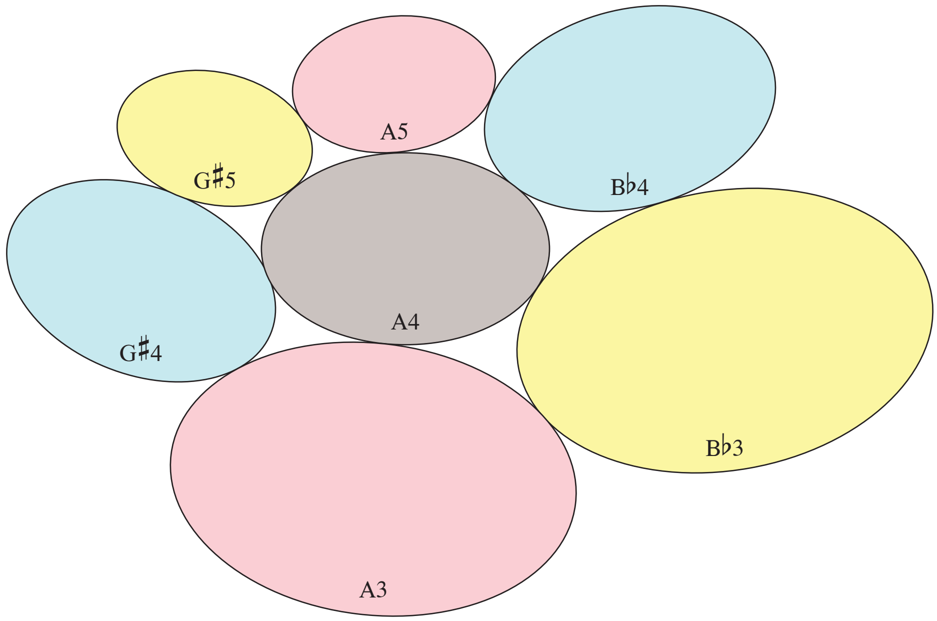


Figure 6. Ovoid notes

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